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Name.....

Reg. No.....

**THIRD SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2024**

Mathematics

MTS 3B 03—CALCULUS OF SINGLE VARIABLE—2

(2019—2023 Admissions)

Time : Two Hours and a Half

Maximum : 80 Marks

**Section A***All questions can be attended.**Each question carries 2 marks.*

1. Differentiate the function  $f(x) = \log\left(\frac{x}{\ln(x)}\right)$ .
2. Find the derivative of  $y = \log(|\sec(x) + \tan(x)|)$ .
3. Find  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{1 - \cos(2x)}$ .
4. Show that  $\cosh^2 x - \sinh^2 x = 1$ .
5. Find  $\lim_{n \rightarrow \infty} e^{\frac{1+n}{1-n}}$ .
6. Determine whether the sequence  $a_n = 1 + (-1)^n / n^2$  converges or diverges. If it converges, find the limit.
7. Express  $.111\bar{1}...$  as a rational number.
8. State the Squeeze Theorem.
9. By using the power series expansion of  $\sin x$ , show that  $\frac{d}{dx}(\sin x) = \cos x$ .
10. Find the Maclaurian series expansion of  $\frac{1}{1+x}$ .
11. Find the rectangular equation of a curve whose parametric equation is  $x = t + 1, y = t^2 - 1$ .
12. Find the equation of the tangent to the ellipse  $x = 3 \cos t, y = 2 \sin t$  at  $t = \pi/4$ .
13. Find an equation of the line that passes through the point  $(-1, 0, 2)$  and is parallel to the vector  $\langle 1, 5, -4 \rangle$ .

**Turn over**

14. Find the equation of the surface  $z = x^2 + y^2$  in cylindrical co-ordinates.  
 15. Find  $r'(t)$  if  $r(t) = 2 \cos ti + 3 \sin tj + 3tk$ .

(15 × 2 = 30 marks)  
 Max. Ceiling : 25 marks

### Section B

*All questions can be attended.  
 Each question carries 5 marks.*

16. Evaluate :

(i)  $\lim_{x \rightarrow 0} \frac{\tan 5x}{\sin 2x}$ .

(ii)  $\lim_{x \rightarrow 0} \frac{x^3 - 3 \sin^2 x}{x^2}$ .

17. Find the derivative of  $y = (\cos x)^x$ .

18. Find  $\int x^2 e^{-x} dx$ .

19. Use the integral test to determine the series  $\sum_1^{\infty} \frac{1}{n^2}$  converge or diverge.

20. Find the interval of convergence and radius of convergence of the power series  $\sum_0^{\infty} \frac{x^{2n+1}}{(2n+1)!}$ .

21. Find the Maclaurian series for  $\frac{1}{\sqrt{1-x}}$  and determine its interval of convergence.

22. Find  $\frac{d^2y}{dx^2}$  for the parametric equation  $x = a \cos t, y = b \sin t$ .

23. Identify and sketch the graph of the surface  $x^2 - x^2 - y^2 = 1$ .

(8 × 5 = 40 marks)  
 Max. Ceiling : 35 marks

### Section C

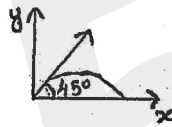
*Answer any two questions.  
 Each question carries 10 marks.*

24. (i) Show that  $\int \frac{dx}{\sqrt{4x^2 - 9}} = \frac{1}{2} \cosh^{-1} \left( \frac{2x}{3} \right), x > 3/2$ .

(ii) Find  $\int_{-\infty}^0 \frac{e^x}{\sqrt{1+e^{2x}}} dx$ .

(iii) Find  $\lim_{x \rightarrow 0} (\tan x)^x$ .

25. (i) Let  $C$  be the ellipse  $r(t) = 3\cos t + 2\sin t$ . Find  $T(t)$  and  $N(t)$  at  $t = \pi/4$ .
- (ii) Find the curvature of the curve  $r(t) = ti + \frac{1}{t}j$  at  $t = 1$ .
26. (i) Find the total arc length of the cardioid  $r = 1 - \cos \theta$ .
- (ii) Find the area of the cardioid  $r = 1 + \cos \theta$ .
27. A shell fired from a cannon, has a muzzle speed of 80 ft/s. The barrel makes an angle of  $45^\circ$  with the horizontal and, the barrel opening is assumed to be at ground level.
- (a) Find parametric equation for the shell's trajectory.
- (b) How high does the shell rise ?
- (c) How far does the shell travel horizontally ?
- (d) What is the speed of the shell at its point of impact with the ground.



(2 × 10 = 20 marks)